1 Gödel's Theorem as a UNNS Constant

A defining feature of recursive systems is the inevitability of undecidable or paradoxical residues once sufficient nesting and self-reference are present. Gödel's incompleteness theorems, classically framed for arithmetic, can be reinterpreted in the UNNS framework as the emergence of an invariant constant: no UNNS with nontrivial recursion can avoid the appearance of undecidable states.

Definition 1.1 (Gödel Constant of UNNS). *Let $\mathcal{U}$ be a UNNS defined by a recurrence of order $D \geq 2$ with coefficient ring inside $\mathbb{Z}[\zeta_n]$, and let $R \in (0, 1]$ be its self-reference rate. The* Gödel constant *$G(\mathcal{U})$ is the assertion that there exists a subsequence of propositions P_n generated within $\mathcal{U}$ that are true but undecidable relative to the recursion rules. Symbolically,*

$$G(\mathcal{U}) \;:\; \exists(P_n) \quad P_n \in \mathcal{U}, \quad P_n \text{ undecidable in } \mathcal{U}.$$

Theorem 1.2 (Gödel–UNNS Constant). *For every nontrivial UNNS with recursion depth $D \geq 2$ and self-reference rate $R > 0$, the Gödel constant holds:*

$$G(\mathcal{U}) \;\text{ is unavoidable.}$$

Moreover, the associated UNNS Paradox Index satisfies

$$\limsup_{n\to\infty} \mathrm{UPI}(P_n) \;>\; 0,$$

so that undecidable residues persist at all depths of recursion.

Proof Sketch. Gödel's original construction encodes a system's own syntactic rules into arithmetical statements, producing self-referential formulas such as "this statement is not provable." In the UNNS setting, the same mechanism is available: once $D \geq 2$ and $R > 0$, recursive nests can encode their own recursion indices. This generates statements whose truth-value cannot be resolved internally, yielding undecidability. The $\limsup$ claim follows from the observation that self-reference contributes a positive lower bound to the numerator of the UNNS Paradox Index, ensuring nonzero paradox residue even as damping factors increase. □

Remark 1.3. *The Gödel constant elevates incompleteness from a peculiarity of arithmetic to a structural law of recursion itself. It asserts that paradox is not an anomaly but a built-in invariant of UNNS substrates. This places it alongside limit ratios, Gauss/Jacobi sums, and FEEC convergence constants as one of the fundamental constants of the UNNS discipline.*